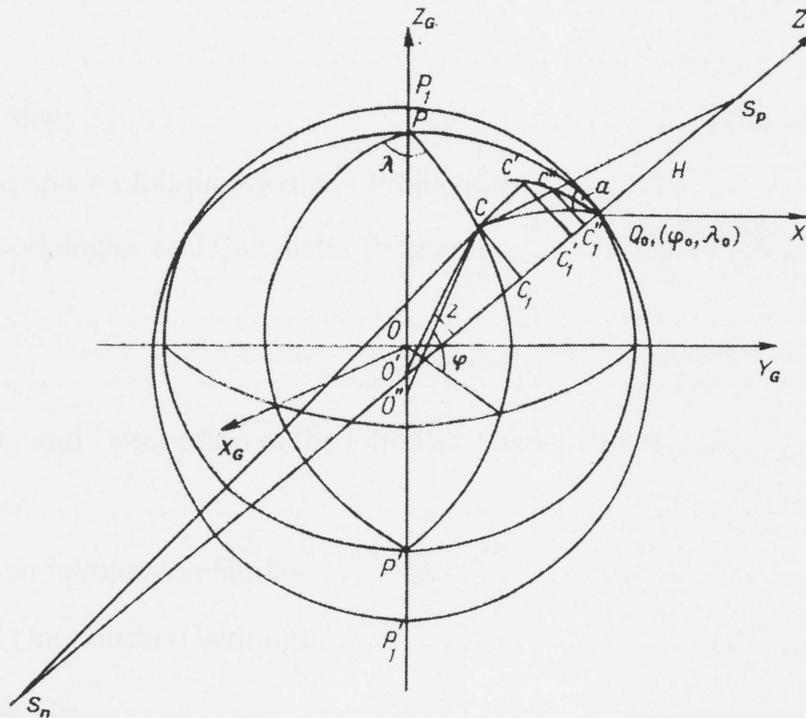


# PROJECTING TIME: JOHN PARR SNYDER AND THE DEVELOPMENT OF THE SPACE OBLIQUE MERCATOR PROJECTION



By

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I would also like to thank some of my colleagues in the Geography and Map Division. James Flatness for first showing me the pile of unopened boxes that comprised the uninventoried Snyder Collection; Dr. John Hébert for suggesting several changes to the paper; Myra Laird for her precise copy editing; Dr. Ronald Grim for listening to and then delivering my paper on John Snyder at the 2003 International Conference on the History of Cartography; and Patricia Molen van Ee for suggesting that I write this monograph and then patiently making it readable. Finally, I would like to thank my wife Dr. Katia Sainson, whose consistent and determined scholarship is always an inspiration for my own.



*Figure 1. Portrait of John Parr Snyder, December 1966*

## PROJECTING TIME: JOHN SNYDER AND THE SPACE OBLIQUE MERCATOR PROJECTION

*John W. Hessler*

We can only substitute a clear mathematical symbolism for an imprecise one by inspecting the phenomena that we want to describe, thus trying to understand their logical multiplicity—not by conjecturing about a priori possibilities.

—Ludwig Wittgenstein<sup>1</sup>

### Introduction

John Parr Snyder (1926–1997) was by all measures one of the most important twentieth century cartographers. The significance and the influence of his work has been compared to that of Gerardus Mercator,<sup>2</sup> a characterization that is well deserved. Snyder's derivation of the equations for the Space Oblique Mercator projection (SOM) and his contributions to the mathematical theory of coordinate transformations rank among the most important developments in the long history of cartographic science.

The SOM projection, like Mercator's, ushered in a new era of cartography, providing the ability to continuously map the earth's surface using satellite data. In 1977, working mostly alone, and as an amateur, Snyder developed the equations for the SOM, which is one of the most complex projections ever devised. Although the SOM was invented and described geometrically in fairly simple terms by Alden Colvocoresses of the United States Geological Survey (USGS), the fact that the projection had to account for the rotation of the earth and the orbital motion of the satellite itself made it mathematically complex. The SOM turned out to be so complex, in fact, that its solution eluded both National Aeronautics and Space Administration (NASA) and the USGS staff for many years before John Snyder's final solution.

The Snyder Family Trust generously donated John Snyder's unpublished papers, manuscripts, and annotated reference library to the Geography and Map Division shortly after his death in 1997. Recently, I have inventoried this collection and produced an annotated finding aid to his manuscripts that is published here for the first time. The importance of this collection for scholars and for the history of cartography is twofold. First, it represents a case study in the history of cartography for a period when computers and satellites were just beginning to transform the discipline into the form that we recognize today. It is the moment in time that saw the invention of new computational and numerical methods, many of which Snyder experimented with in his manuscripts and used in innovative ways in his projection research. He performed much of his most creative work on early programmable calculators, struggling with problems of memory allocation and calculating speed. The programs that he wrote for these calculators were stored on small magnetic strips, shown with his calculators in Figure 2.

In order to preserve the program code in a way that would make the programs useable to future researchers, I first had to learn the programming language for the long obsolete Texas Instrument 59 and 56 programmable calculators, and then search through Snyder's mathematical notes and manuscripts for clues to his programming techniques. Using retrograde analysis and backward induction, I was able to reconstruct many of his programs. Although this process posed challenging problems, the code of these programs has yielded deep insights into Snyder's thinking and the difficulties inherent in programming these early devices.

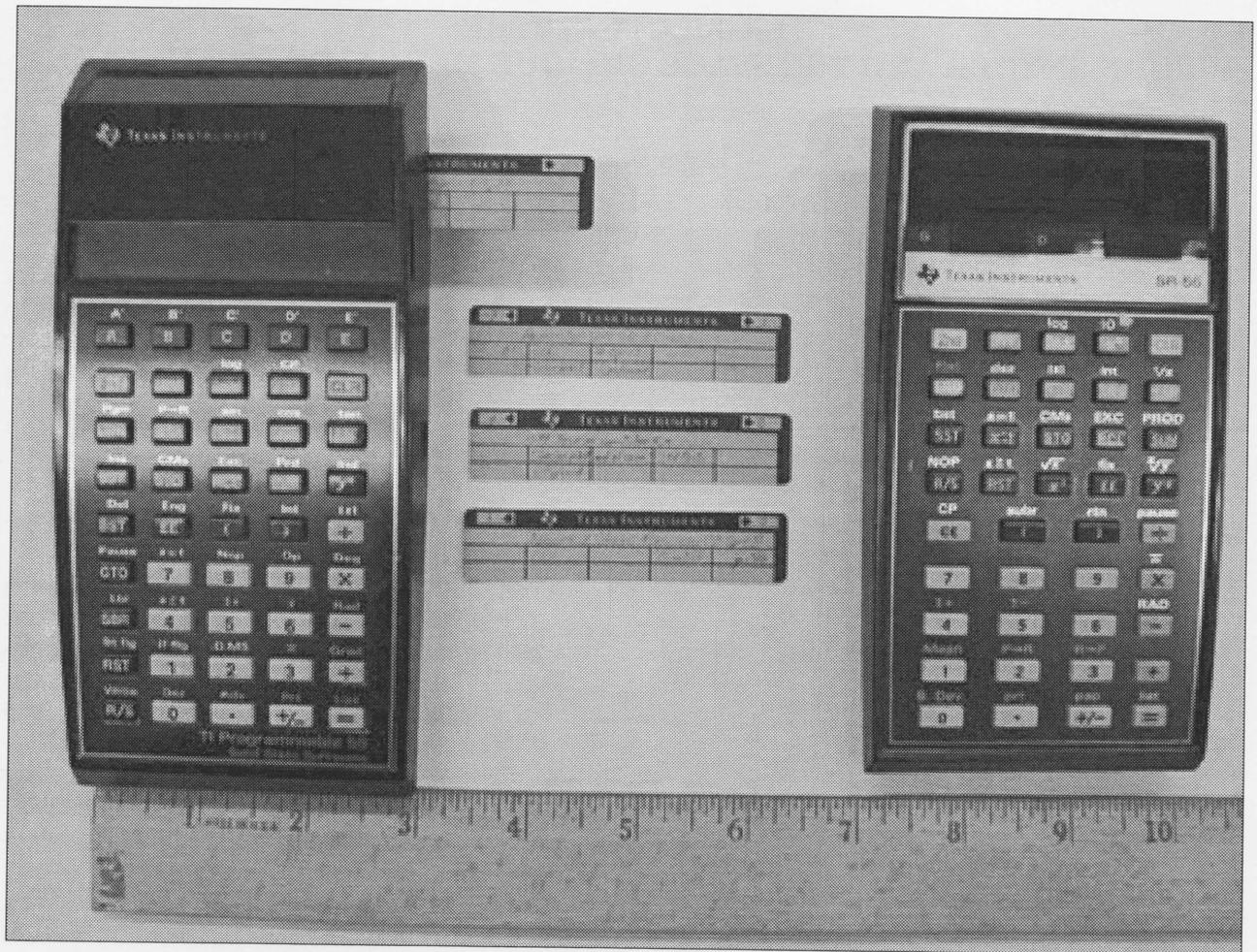


Figure 2. Magnetic programming strips and Snyder's TI-59 and TI-56 calculators

Second, and more generally, the Snyder collection challenges our conception of the content and scope of the historiography of cartography. Most of the writing and models that currently define this historiography are built around a priori definitions of maps as printed artifacts that are to be contextualized as to their use and production. These assumptions, and the knowledge base of the historians who make them, are increasingly insufficient as we begin to examine the cartography of the second half of the twentieth century and beyond. The mathematical and technical knowledge needed to preserve and interpret the Snyder papers gives us a glimpse into the future of cartographic history. This history will be comprised not merely of artifacts, but rather of complex mathematics and computer programs, whose visualization as printed maps is secondary compared with the conceptual and theoretical knowledge that produced them. Unless cartographic historians recognize this and begin to think of maps in terms of other formats than those currently most familiar to the librarian, collector, and paper conservator, much of the history of modern cartography may be lost.

Mathematical cartography, more than anything else, was John Snyder's life, his refuge, and his inspiration. In order to understand the man, the legacy that he left, and the future research that his work spawned, it is necessary to discuss his notion of cartography and to look deeply into his mathematical and programming methods. Some of these methods cannot be discussed without entry into some mathematical and technical details, but I have tried to keep these to a minimum and have provided a large bibliography in those areas where the reader might wish

to explore further. Snyder was not only a cartographer, although that is our focus here, but also an engineer, a lover of music, and a Quaker. He was deeply concerned about civil rights and the injustices of the human condition. He was humble, quiet, and unassuming. He was a husband and a father and to some of us who marvel at the mathematical insights present in his work, he is a legend.

This paper consists of four parts. The first is a brief biography of John Snyder's life focusing on his interest in maps and his involvement in cartographic history up to the time when he began his work on the Space Oblique Mercator Projection. Snyder's early intellectual development was full of mathematics and some of the earliest papers in his collection are projection notebooks dating from the time when he was sixteen years old. In this sense, he truly was a prodigy. The second part consists of a history of the development of the Space Oblique Mercator Projection and Snyder's derivation of its analytical equations. The third part concentrates on Snyder's mature mathematical work and the programming methodologies that he employed throughout his career. In this discussion I rely heavily on his unpublished works and correspondence. I will explore his working methods in an effort to explain the problems he solved and to consider why his solutions are so important to cartographic history. The final part consists of an annotated inventory of his papers and manuscripts that currently reside in the Geography and Map Division.

## Early Cartographic Steps

John Snyder's interest in map projections began at an early age. Sometime in 1942, when he was just sixteen years old, he began collecting in small notebooks various geographical, mathematical, and astronomical facts that interested him and that he thought were worth copying. The notebooks are extremely detailed and although they contain no original information, they are graphic examples that show Snyder as a talented draftsman and display his early interest in cartographic projections. Two of these notebooks remain extant in the Snyder collection; examples of their contents are shown in Figures 3 and 4.

The notebooks contain a great deal of mathematical subject material ranging from simple trigonometric identities through more advanced differential and integral calculus, and solid and plane geometry. The pages shown in the figures are just two of many that contain notes and drawings about or relating to the subject of map projections. Both of these figures show that Snyder was not only drawing the various projections, but was also concerned with details of their construction, a concern that foreshows his later work on the subject. The collection contains numerous examples of Snyder's early and adolescent cartographic experiments including his handmade icosahedron globe shown in Figure 5.

John Snyder's fascination with maps and projections continued through his high school years, and later, at a less active level, during college at Purdue University, where he chose to major in chemical engineering, not cartography. It was later, during his time as a professional chemical engineer at CIBA-Giegy Corporation in New Jersey during the late 1960s and 1970s, that he began seriously pursuing his cartographic interests.

In 1963 he submitted to Rutgers University Press a book entitled *The Story of New Jersey's Civil Boundaries, 1606–1964*, which he had been working on for several years. The work was rejected by Rutgers but was finally published in 1969 by the New Jersey Bureau of Geology and Topography. The book stands out as the first true modern survey of the history of boundaries in New Jersey and was an attempt by Snyder to reconstruct earlier works such as Thomas Gordon's *1834 Gazetteer of the State of New Jersey*, a copy of which he found at the Madison, New Jersey, public library. Snyder's second book also concentrated on the history of mapping New Jersey. Entitled *The Mapping of New Jersey—the Men and the Art*, it was published by Rutgers University Press in 1973. The book itself is a standard cartographic history and begins with the cartography of North America before 1750 and extends its coverage into the twentieth century. Lawrence



Spellman, the map curator at the Firestone Library of Princeton University, reviewed the book in the first issue of the *American Cartographer*. Spellman wrote,

Engineer-cartophile Snyder explores these facets [New Jersey mapping history] . . . in a fact filled volume which will fascinate cartographers. He attains a fine balance of technical knowledge with a scholarly historical background, showcasing both in an energetic and engaging prose style manifested too infrequently in works of this genre.<sup>3</sup>

During this period Snyder also wrote additional historical works including several articles and a pamphlet on the mapping of New Jersey during the Revolutionary War. As an outgrowth of his research on mapping New Jersey, Snyder published a short history of the Erskine-DeWitt maps in 1979. The Erskine-DeWitt maps are a set of 365 road maps prepared for the Continental Army under George Washington, of which 300 are still extant. Northern New Jersey was geographically and operationally the focus of the surveys, but roads to Connecticut, Albany, the Finger Lakes, and Yorktown were also included. The New York Historical Society owns the original manuscript maps and at the time had no plans to publish them. Snyder studied and indexed the maps with the hope of preparing a critical edition with reproductions of the manuscripts. When the New York Historical Society denied him permission to publish his edition, he had to content himself with a detailed historical study. The study looks deeply into the accuracy of the surveys and Snyder comments on the projections and the meticulousness of the surveyors, Robert Erskine and Simeon Dewitt.

According to Snyder's correspondence, from the early 1970s, after the publication of his second book, he wanted to expand his knowledge of cartography and began research on a third book, this time on the history of projections. During his research Snyder found that the book-length literature on the subject of projections was extremely limited and really consisted of only two extensive reviews done in the nineteenth century, the studies of Marie Armand Pascal d'Avezac in 1863 and Adrien Adolphe Charles Germain in 1865,<sup>4</sup> annotated copies of which are in the Snyder Collection. Although he spent many years involved in research, Snyder never published this early work on the history of projections. The revised manuscript survives and shows substantial changes and revisions from its first draft. The annotations and changes made to the manuscript reflect Snyder's growing dissatisfaction with the published descriptions of the projections he was finding in the literature, many of which contained errors or did not provide sufficient derivations to permit the projection's accurate reconstruction.

Snyder sent his first substantive article on projections to the editor of the *American Cartographer*, then Arthur Robinson, in March 1976.

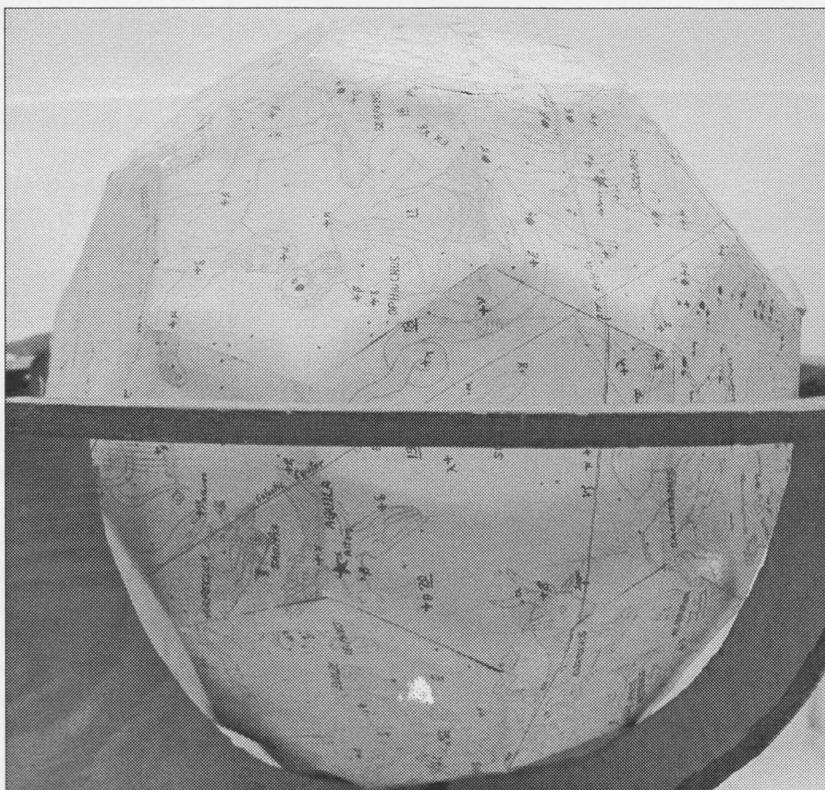


Figure 5. Polyhedral icosahedron globe

Consisting of a subset of the work he had done for his book on projections, it is a comparison of pseudocylindricals, which are characterized by straight horizontal lines for parallels and usually equally spaced curved meridians of longitude. The pseudocylindricals had previously been described in a wide range of books and periodicals, but Snyder set out to correct errors in the published descriptions and to provide correct mathematical formulas that would allow direct coordinate calculations for this class of projections. The paper is an example of what would become a trademark of Snyder's later works, namely the accurate compilation, derivation, and correction of projection literature. The paper was accepted and published in 1977.<sup>5</sup>

Snyder's early correspondence with both Waldo Tobler and Arthur Robinson is one of the most interesting features of his collection and can be read as a representation of his growth as a cartographer. The earliest correspondence that survives from mid-1976 shows Snyder tentative and humble in his approach to the giants in the field he was trying to enter. His letters to Robinson are more extensive than those to Tobler and cover a longer period of time, from 1976 to shortly before Snyder's death in 1997. The subjects range widely from computer projections to the issue of the Peters Projection and the politicization of cartography. Snyder's letters to Tobler are more mathematical and more oriented toward projections, beginning in October 1976 and continuing sporadically into the 1980s. One particular letter from Tobler will turn out to be the pivotal element that convinced Snyder to attempt what became his most important work, the derivation of the equations for the Space Oblique Mercator Projection.

## Development of the Space Oblique Mercator Projection

The discussion of John Snyder's derivation of the transformation equations for the Space Oblique Mercator Projection must necessarily begin with the launch of ERTS, Landsat-1, in the summer of 1972. Landsat-1 orbited the earth with a satellite ground track that proceeded essentially along an oblique circumference of the earth, approximately nine degrees off the poles. As Dr. Alden Colvocoresses (Colvo), cartographic coordinator for earth satellite mapping at the USGS, realized early that Landsat-1 had the capabilities to provide continuous mapping data, he began to imagine an entirely new type of map projection. Map projections are typically static pictures of a stationary earth. Colvo's conception for a new family of projections that could map a continuous data stream required the additional dimension of relative motion; therefore, time had to be considered as a mapping parameter.

Colvo described the geometry of this new projection in an extremely important article in 1974.<sup>6</sup> He conceived of a cylinder defined by a circular orbit (Figure 6) with the projection surface tangent to the spheroid of the earth. A photograph of Colvo's original model is shown in Figure 7. His initial description accounts for four principle motions: the satellite's scanner sweep across the earth's surface; the satellite's orbit; the earth's rotation; and the earth's orbital precession. To keep these motions from distorting the image, the cylindrical surface of the projection was made to oscillate along its axis at a compensatory rate that varied with latitude. No projection with this geometry had ever been conceived before and both NASA and the USGS were extremely skeptical that its analytical solution could be found.<sup>7</sup>

In mathematical terms a projection is defined as a one-to-one correspondence between points on the scaled down model of the earth or datum sphere and a flat plane. Since the surface of a sphere cannot be laid flat on a plane without introducing distortion, a fact proved by Karl Friedrich Gauss in his *Disquisitiones Generalaes circa Supericies Curvas* of 1827, a set of transformation equations is necessary to describe the relationship of the sphere to the flat plane. These equations describe the nature of the correspondence between the latitude and the longitude on the earth's surface and the distances and angles on the flat plane. The distortion that is introduced by this projection process may be in area, length, angle, or shape and are all defined by the specific projection equations.

In the case of the Space Oblique Mercator, Colvo could accurately describe the projection's

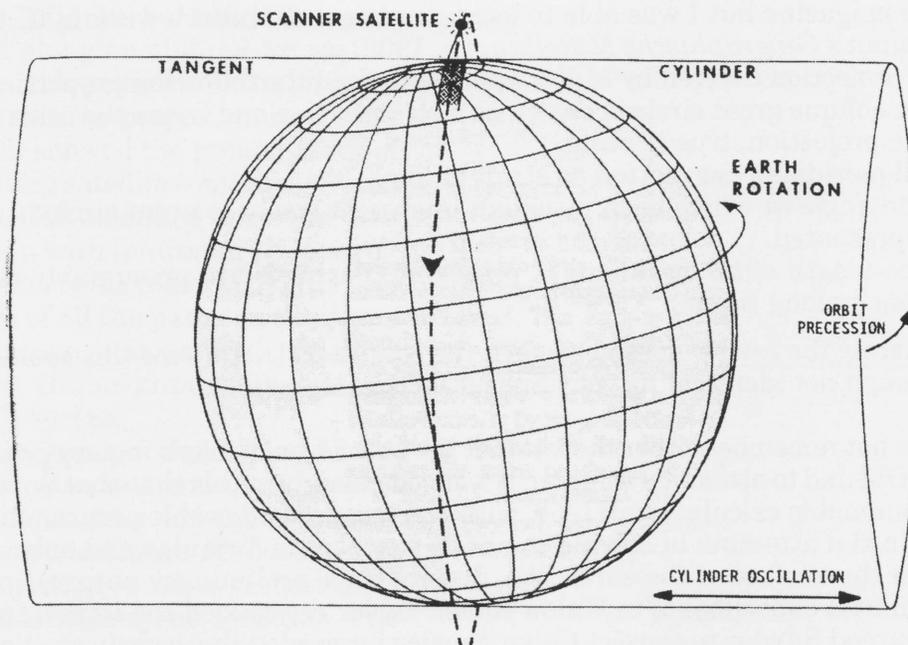


Figure 6. Colvo's geometric conception of the Space Oblique Mercator projection

shape and mapping parameters, but without the transformation equations the real character of the projection and its level of distortion remained unknown. Until these equations could be specified, the ability to continuously and conformally map the surface of the earth using satellite data remained impossible.

Several years after Colvo's initial description of the projection at a conference at Ohio State University in 1976 entitled "The Changing World of Geodetic Science," John Snyder heard a paper presented by Colvo on his new projection.<sup>8</sup> During his presentation, Colvo lamented the fact that no one had yet been able to solve the equations for the SOM. According to notes in Snyder's manuscripts, he showed some interest in the paper but proceeded no further. Then in April 1977 Snyder received a copy of a letter sent to Colvo by Waldo Tobler calling Colvo's attention to a German paper that he thought might help with the solution of the SOM. The letter addressed to Colvo begins,

Recently I came across a reference to a paper in the *Sddeutsche Techniker Zeitung*, Munich 1905, by one Franz Muller, an engineer. In tracking down this I found that our library does not

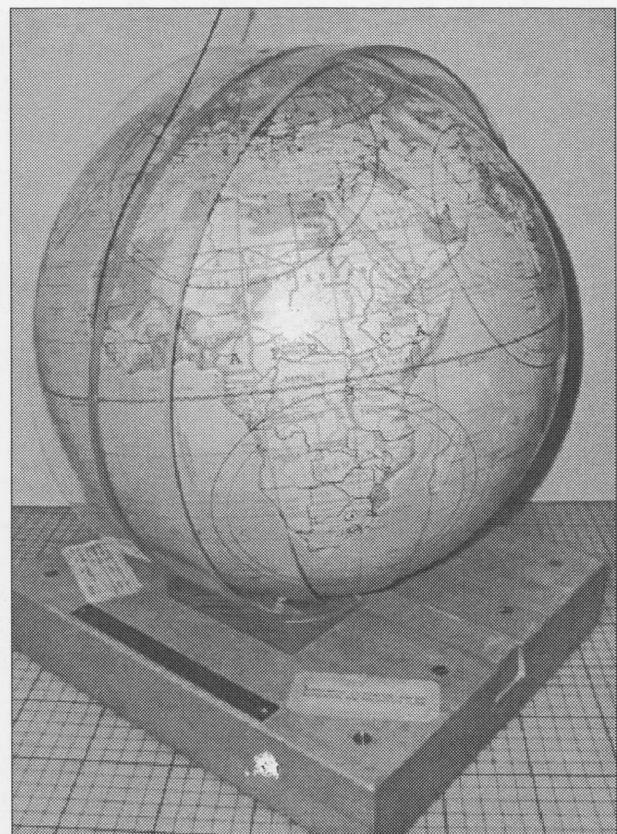


Figure 7. Original USGS model of the SOM projection showing satellite orbit track and oscillating cylinder

have this magazine but I was able to locate a review of Muller's work by E. Hammer in *Petermann's Geographische Mitteilungen*, 1906, pp. 92–94.

The projection derived by Miller apparently had the following properties:

1. An oblique great circle is drawn as a straight line and forms the central axis of the projection, true to scale.
2. All meridians are plotted as straight lines.
3. The angle at which each meridian intersects the base great circle is correctly represented.
4. The length of all meridians is preserved. Parallels are presumably then positioned along these.

Unfortunately the review does not repeat the equations [ . . . ]. To me this sounds rather like, although not identical to, your Space Oblique Mercator.<sup>9</sup>

Colvo does not remember either the letter or the paper,<sup>10</sup> but Tobler's inquiry prompted Snyder to revisit the SOM and to attempt a solution. It was coincidentally at this time that Snyder purchased his first programmable calculator, a TI-56, with one hundred allowable programming steps.

Snyder's initial attempts in solving the SOM were crude formulas and only calculated for a few points on the surface of the earth. Yet, despite their preliminary nature, Snyder decided to send these initial experiments to Waldo Tobler. Tobler recognized the importance of the calculations and urged Snyder to contact Colvo. Snyder forwarded these early studies of the SOM to Colvo, who was encouraged, and sent back several suggestions on the geometry of the projection. Snyder, while in close contact with Colvo, continued to improve his derivations and in August 1977, just five months after he began work, he sent to Colvo a set of completed equations for the SOM treating the earth as a perfect sphere.

To discuss the Space Oblique Mercator is to speak of one of the most complex projections ever devised and requires a few technical details. The SOM (Figure 8) is part of the family of conformal projections and was originally described as an offshoot of Mercator's projection in its

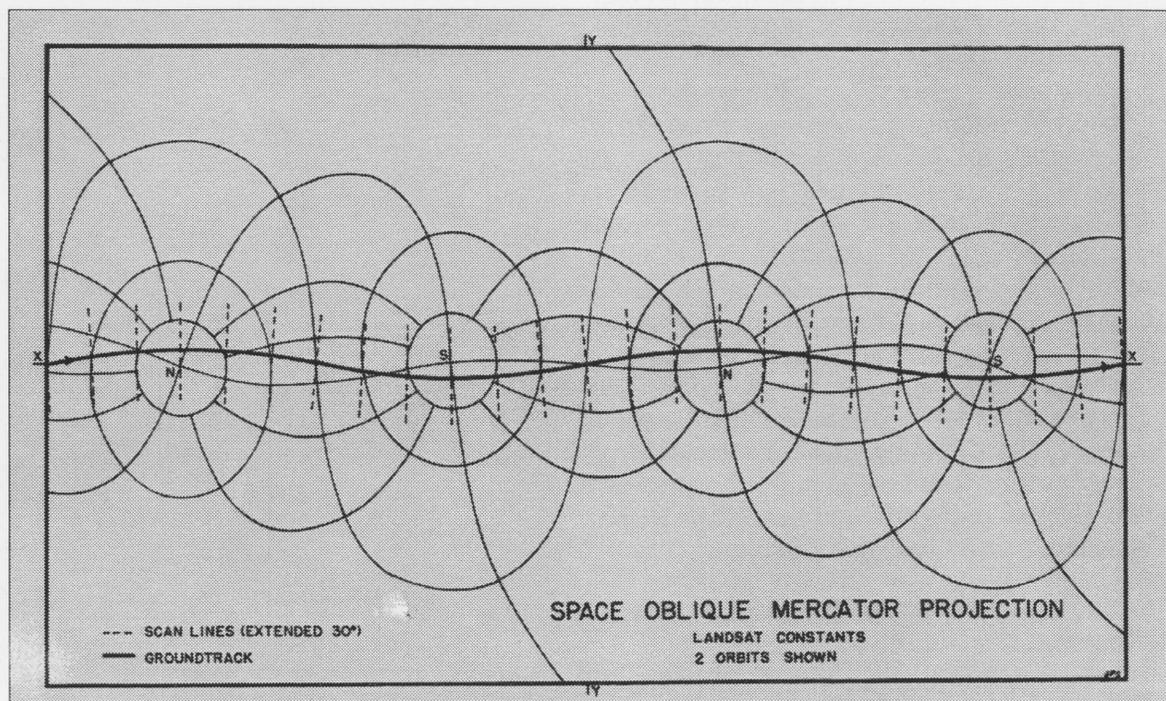


Figure 8. Space Oblique Mercator projection with two orbits

oblique form (Figure 9c) because Landsat-1 orbited along an oblique circumference to the earth's poles. Until the creation of the SOM no map projection had been devised which showed the ground track of an earth orbiting satellite continuously and true to scale. Colvo defined the SOM as a conformal map with minimum scale error, a map projection being conformal if the projected images of all the paths on the plane of the map intersect at an angle equal to that between the original paths on the actual earth's surface.

Snyder began his derivation of the SOM with the simplest and most general form of the equations for the Mercator projection shown in Figure 10. The regular Mercator projection consists of meridians projected onto a tangent cylinder. The parallels cannot be projected directly onto the cylinder but must be spaced according to the equation for  $y$  in Figure 9. Snyder developed a group of intermediate transformation equations and modified them to account for the motions involved in mapping the satellite's path or ground track along the surface of the earth.

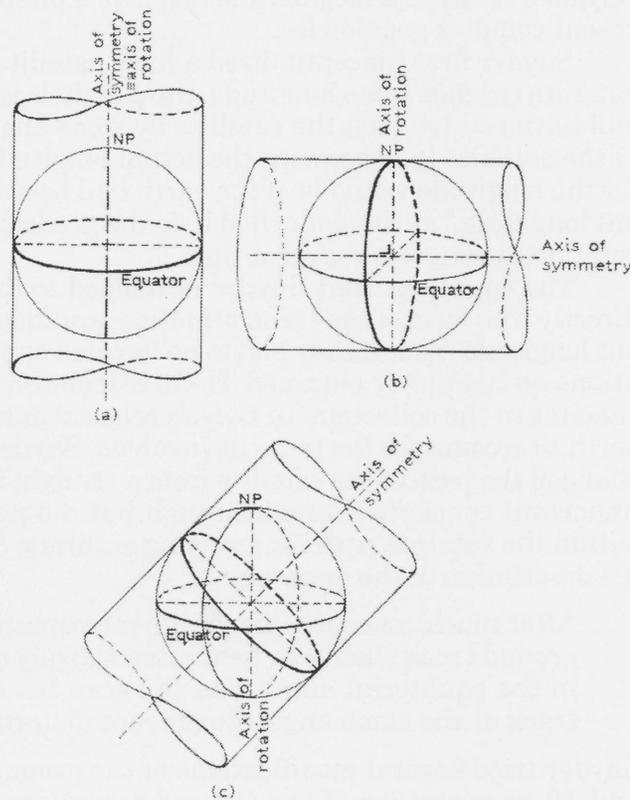


Figure 9. Schematic of the projection surface for cylindrical projections

$$\begin{aligned}
 x &= a\lambda \\
 y &= \frac{a}{M} \log \left[ \tan \left( \frac{\pi}{4} + \frac{\phi}{2} \right) \left( \frac{1 - \epsilon \sin \phi}{1 + \epsilon \sin \phi} \right)^{\epsilon/2} \right] \\
 &= \frac{a}{M} \log \tan \left( \frac{\pi}{4} + \frac{\chi}{2} \right) = \frac{a}{M} \log \cot \frac{z}{2};
 \end{aligned}$$

where  $\phi$ ,  $\lambda$  are the geodetic latitude and longitude, respectively,  $\epsilon$ , is the eccentricity of the meridian ellipse,  $M$  is the modulus of the common logarithm.

Figure 10. General equations for Mercator projection

An easy way to visualize this ground track is to imagine the shadow of the satellite cast upon the surface of the earth as it orbits. His transformation equations modified the geometric constraints of the problem by substituting an alternate set of latitudes and longitudes measured from a transformed equator that was at an oblique angle to the earth's actual equator. To derive the curve for the actual ground track, Snyder used these transformed equations and accounted for the earth's motion by introducing more complex differential and integral calculus to determine the instantaneous slope of the ground track relative to the moving longitude and latitude of a rotating earth. Simply, he found a method for determining how the longitude and latitude changed during some arbitrary period of time relative to the satellite's position.



PROJECTING TIME—JOHN PARR SNYDER

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*Figure A. Early projection notebook page showing azimuthal projections*

*Figure B. Early projection notebook page showing azimuthal projections*

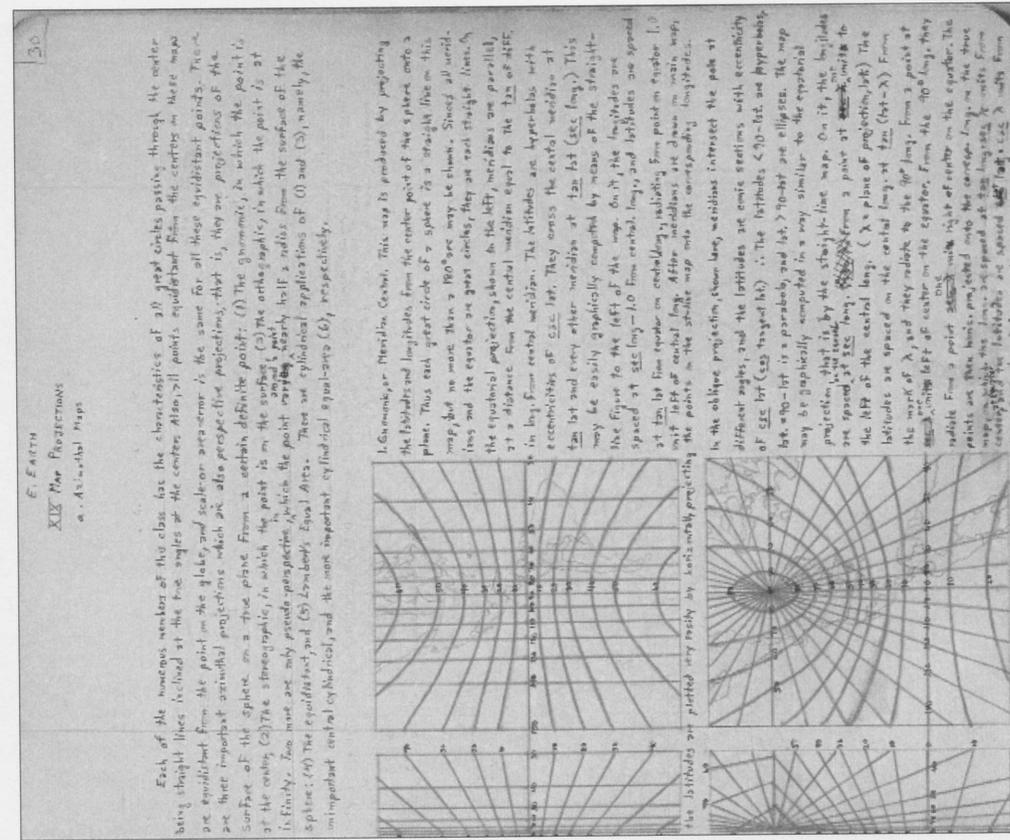
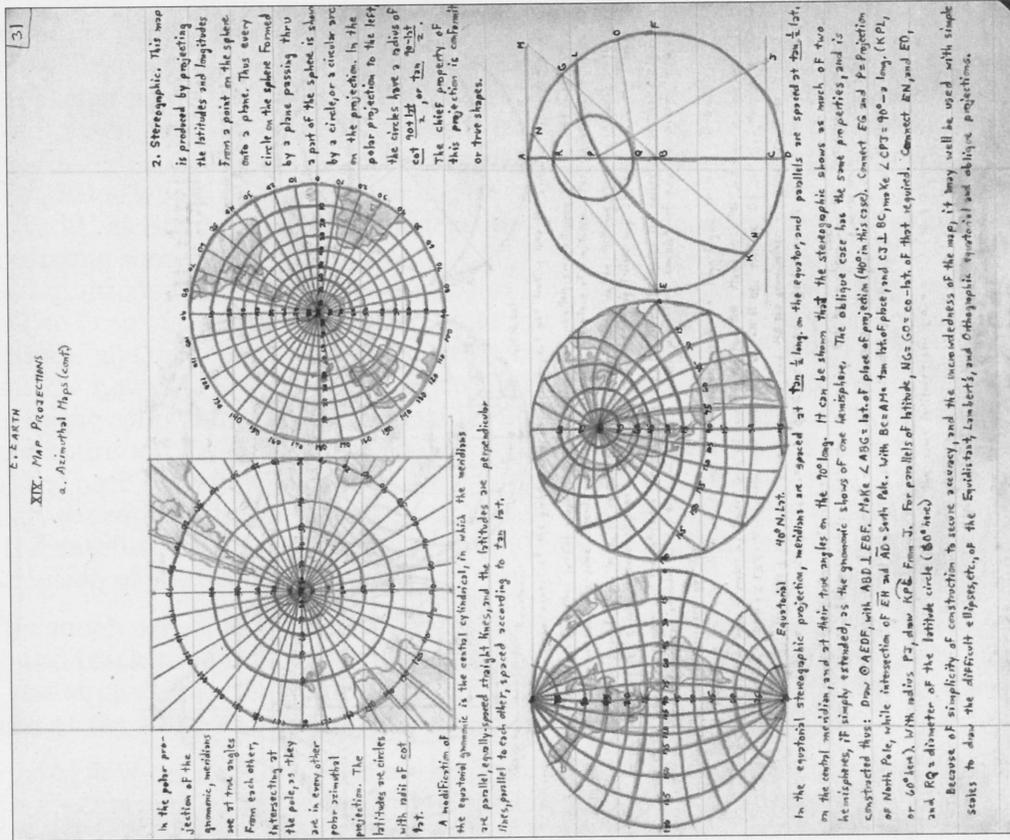


Figure B

Figure A

### Angular Distortion Spherical Case

Space Oblique Mercator Projection  
Snyder's TI-59 Program

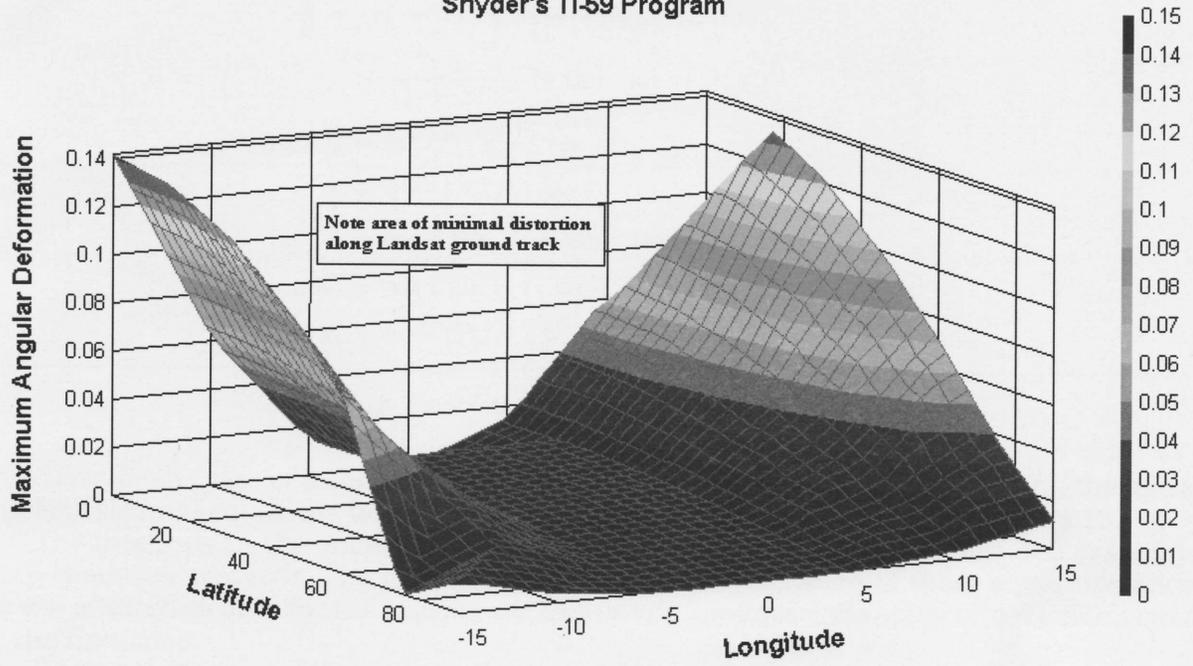


Figure C

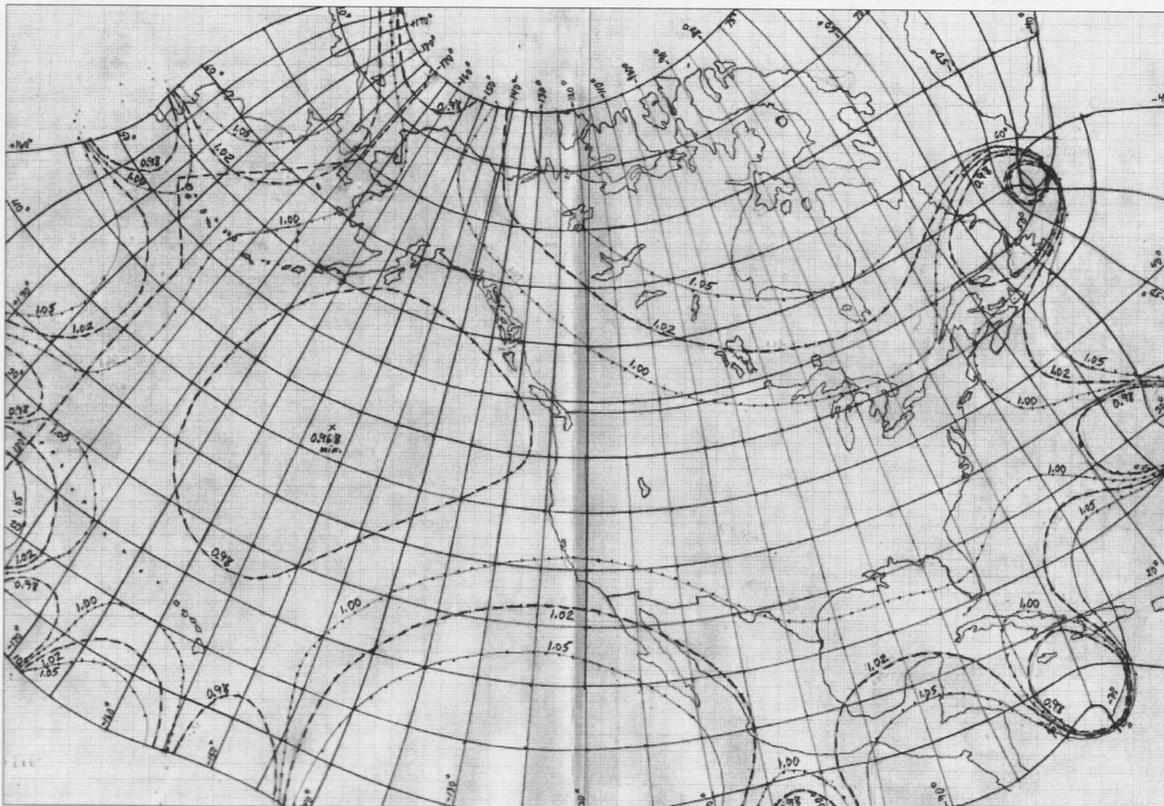


Figure D

*Figure C. Reconstructed program results for angular distortion of SOM*

*Figure D. Smoothed distortion plot for GS50 projection*

$$\frac{x}{R} = \int_0^{\lambda'} \frac{H - S^2}{(1 + S^2)^{1/2}} d\lambda' - \frac{S}{(1 + S^2)^{1/2}} \ln \tan \left( \frac{1}{4} \pi + \frac{1}{2} \phi' \right)$$

$$\frac{y}{R} = (H + 1) \int_0^{\lambda'} \frac{S}{(1 + S^2)^{1/2}} d\lambda' + \frac{1}{(1 + S^2)^{1/2}} \ln \tan \left( \frac{1}{4} \pi + \frac{1}{2} \phi' \right)$$

where  $S = (P_2/P_1) \sin i \cos \lambda'$

$$H = 1 - (P_2/P_1) \cos i$$

$$\tan \lambda' = \cos i \tan \lambda_t + \sin i \tan \phi / \cos \lambda_t$$

$$\sin \phi' = \cos i \sin \phi - \sin i \cos \phi \sin \lambda_t$$

$$\lambda_t = \lambda + (P_2/P_1) \lambda'$$

Figure 13. Final form of the SOM equations spherical case

equations for the case of a spherical earth and a spherical satellite because the mathematics is simpler. His final equations for the spherical case of the SOM are shown in Figure 13.

The integrals in the equations cannot be solved analytically and hence had to be solved using computer methods. The iteration and numerical integration of these equations allowed the use of satellite data for the continuous and nearly conformal mapping of the earth's surface for the first time.

During the course of his work on the spherical case outlined above, Snyder recognized that it would not be sufficiently accurate for practical use in mapping and that he would have to develop more complex forms of the SOM equations. In the end he worked through four progressively more complex sets of the SOM equations:

1. Assumes the earth to be a perfect sphere and the satellite orbit to be circular.
2. Assumes the earth to be an ellipsoid and the satellite orbit to be circular.
3. Assumes the earth to be an ellipsoid and the satellite orbit to be elliptical with an eccentricity of less than 0.05 or less
4. Assumes the earth to be an ellipsoid and the satellite orbit to be an ellipse with any eccentricity.

As accurate as Snyder's formulas were for the spherical case of the SOM, errors of over one-half of a percent still persisted because of the assumption that the earth was a perfect sphere. In maps of large areas, errors coming from the projection process far exceed those due to assumptions regarding the shape of the earth, but for accurate topographic mapping of small areas or strips, the use of an ellipsoidal earth is essential for accurate representations. The process of representing the earth as an ellipsoid greatly complicates the mathematics involved. Snyder hoped to shortcut his derivations by using the classic work of Martin Hotine on the Ellipsoidal Oblique Mercator Projection.<sup>12</sup> Hotine discusses in detail the fact that the ellipsoidal version of the oblique Mercator projection cannot be derived exactly. Instead, Hotine derived an approximation using what he termed an "aposhere," which is a surface of constant curvature, tangent to the ellipsoid at a chosen point. Hotine's methods, while approximate, do yield an exactly conformal projection, and Snyder assumed this provided a logical link for the development of the ellipsoidal case of the SOM. Unfortunately for Snyder, when he introduced his equations for the revolving satellite the errors in Hotine's approximations grew unacceptable. Snyder's curved ground track presented additional problems in adopting Hotine's solutions, and he soon found this route mathematically intractable. Snyder returned to the more difficult route he had previously tried to avoid, and derived the geometry he needed from the basic principles and mathematics of ellipsoids. Using this geometry he derived the more complex cases of the SOM.

In late August 1977 after Snyder's solution of the first two cases of the SOM, Colvo gathered a group of scientists and engineers at USGS headquarters in Reston, Virginia, to discuss Snyder's solution. Among those attending was Dr. John Junkins of the University of Virginia, an expert in the mathematics of orbital dynamics and the person originally given the contract by NASA and the USGS to solve the SOM equations. Junkins' solution<sup>13</sup> was more complex and more general than Snyder's, but it contained a flaw that gave distortion errors twenty times those of Snyder's equations and was, therefore, not useful for mapping applications. Later both Snyder and Junkins, using the computers at the University of Virginia and the USGS, would search for the flaw in the general form of the equations, but unfortunately it was never found.

After his solution of the SOM, Snyder was hired by the USGS as a projection specialist, and in 1978 he received the John Wesley Powell Award from that agency. As the text of the award indicates, "he provided the long sought after link by which Earth surface data obtained from orbiting spacecraft can now be transformed to any of the common map projections. This is an essential step in automated mapping systems which we see developing." The Associated Press picked up the story from a USGS press release and dozens of newspapers carried the story of how an amateur, working mostly alone and with a pocket calculator, solved one of the most important cartographic problems of the century. The *New York Times* sent their senior science reporter, John Noble Wilford, to interview Snyder; Wilford subsequently wrote an article entitled, "Mapping Hobbyist Sharpens Images of the Earth from Space."<sup>14</sup> Snyder's equations in one form or another have been used in all Landsat and other earth mapping satellites and continue to be utilized in most current earth mapping systems.

## Mathematical Methodologies and Calculator Programs

The Snyder collection contains computer programs that he wrote in a variety of languages (i.e., Fortran, Assembly, Basic, and pseudo-C). However, a large portion of John Snyder's most important mathematical work was created and tested using numerical techniques that he developed for the T-56 and T-59 calculators. Both of these calculators were manufactured by Texas Instruments Corporation and are part of the first generation of affordable programmable calculators. They represented a breakthrough in portable computation and were the mainstays of engineering calculation during the 1970s. Both of Snyder's original calculators are part of the collection in the Geography and Map Division. (See Figure 2.) Snyder chose these TI products because he liked the straightforward nature of their algebraic entry and programming as opposed to the reverse logic employed by the comparable Hewlett-Packard calculators.

One advanced feature of the TI-59 calculator that set it apart from others available at the time was its ability to store programs on the small magnetic strips that are also shown in Figure 2. Over three hundred and fifty of these strips programmed by Snyder remain extant in his papers. The ability to store programs externally freed the user from having to re-enter the program codes each time a calculation was performed. Because these strips are a magnetic medium and inherently unstable, and because much of Snyder's most important work was in fact written on them, it was necessary for me to find a method for reconstructing the code they contained.

The preservation of the program code presented both technical and mathematical challenges. Snyder's mathematical notes provide explicit clues to his programs in only a few cases. This fact required me to learn the now obsolete TI-59 programming language and to search through his notes for any additional clues to his programming techniques. Using these notes and the explicit instructions he left for a few of his programs, I have been able to reconstruct ninety-nine of the extant programs, including the most important examples that are associated with the Space Oblique Mercator Projection.

Snyder employed many novel methods and invented some new numerical techniques when programming these devices. A problem he continually encountered was that of memory allocation. The TI-59 had a combined program limit of nine hundred and sixty steps, which was a vast

improvement over the one hundred-step limit of the TI-56. What this meant for him in practical terms is that for a calculation to be performed a balance had to be struck between the number of data positions or the numerical values being entered, calculated, or stored and the number of steps in the program being executed. This balance is shown graphically in Figure 14. He searched for algorithms that might help him compress the number of steps he needed to use and translated many newly developed procedures, such as those found in Donald Knuth's now classic book, *The Art of Computer Programming* (1969), into the TI language.

The programs that have been reconstructed so far can be run on an emulator that mimics the TI-59 operation and allows the three-dimensional display of the program's results. The output for the reconstructed program that calculates the distortion of the spherical case of the SOM is shown in Figure 15, which displays the low error for the projection, shown as the trough at the bottom of the graph. In solving the SOM, Snyder wrote programs to solve Fourier transforms and to perform Simpson's rule integrations, procedures that for 1977 must be considered fairly advanced home computation. The program reconstructions and the speed of the emulator allow the modern researcher access to Snyder's programs as well as a firsthand view of his methods.

A detailed examination of Snyder's development of the GS50<sup>15</sup> Projection provides an example of the type of programs and derivations that make up the Snyder collection and furthers our insight into his working methods. Snyder developed the GS50 Projection in the early 1980s to be a low-error conformal projection showing all fifty of the United States and the ocean passages between them. His new projection had a scale distortion of plus or minus two percent and was considerably less than the plus twelve to minus three percent range of the then existing USGS map of the fifty states.

Snyder used a technique of complex-algebra polynomial transformations, which had been known for quite some time, to generate conformal map projections with less scale variation. This transformation is based on the transfer of coordinates from one projection to another, in other words, taking an already projected map and re-projecting it in order to improve its accuracy. Polynomial transformations of the projection coordinates of one map to another can be used under certain conditions to alter the distortion patterns and to obtain a more accurate map of a specific area. Polynomial transformations can also be used to reduce computation time if the

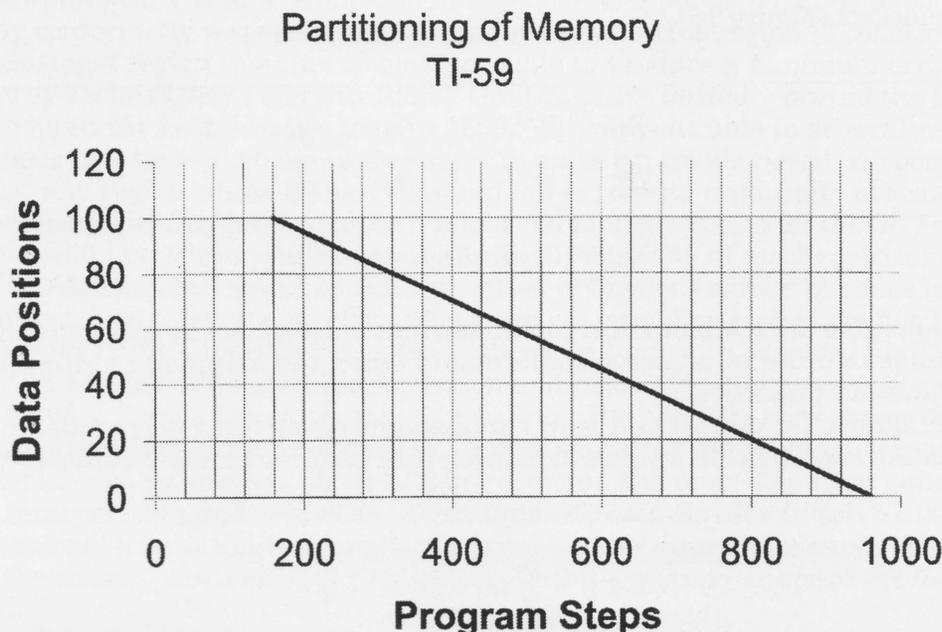


Figure 14. Partitioning of memory for TI-59 calculator

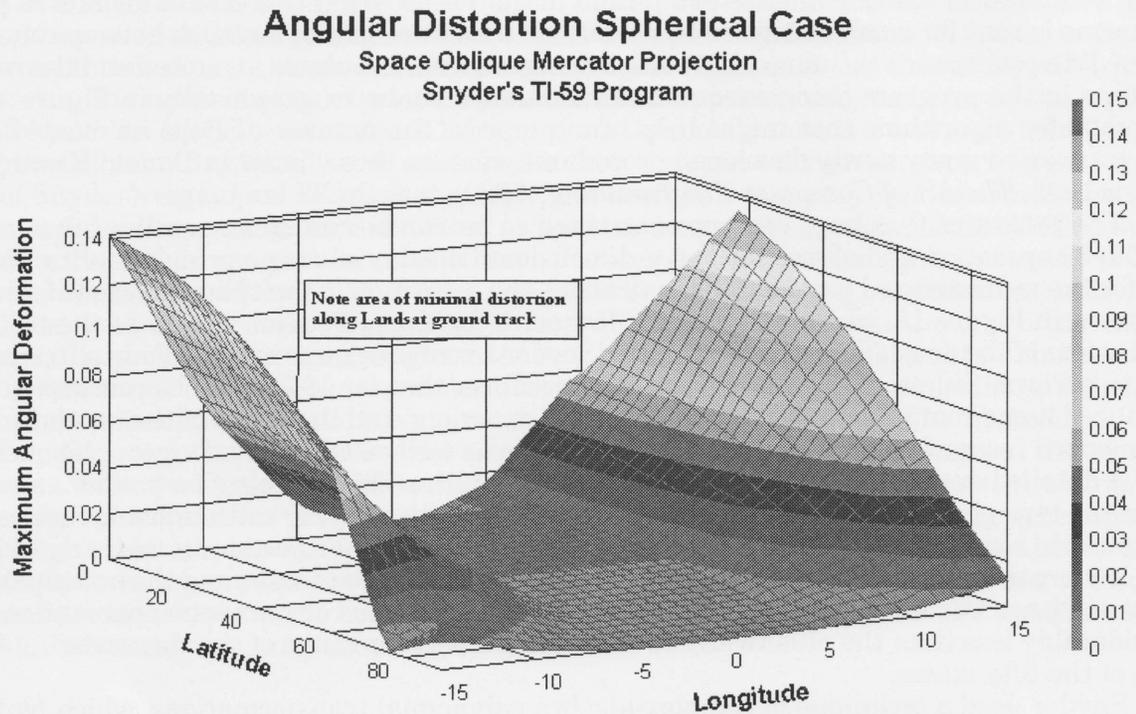


Figure 15. Reconstructed program results for angular distortion of SOM

number of coordinates to be transformed is very large, or to transfer data between maps with incomplete definitions of projection parameters.

The general conditions for a conformal transformation of one map to another, or one set of coordinates to another, are given by a set of partial differential equations known as the Cauchy-Reimann equations (Figure 16).

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Figure 16. Cauchy-Reimann conditions

These equations give the mathematical conditions that must be met by any set of conformal projection formulas in order to preserve conformality when the mapping coordinates are transformed into another projection.

In 1932 Ludovic Driencourt and Jean Paul Laborde developed a polynomial that meets the Cauchy-Riemann conditions in a series expansion that involves complex numbers<sup>16</sup> (Figure 17).

$$x + iy = \sum_{j=1}^n (A_j + iB_j) (x' + iy')^j,$$

Figure 17. Driencourt polynomial expansion

The details of the expansion are beyond the scope of this paper, and the reader is referred to Driencourt and Laborde (1932) for details. It is enough to understand that the advantage of expansions like this one is that the lines of constant scale on the map may be made to follow a variety of patterns instead of simply following the great and small circles of the earth as on common conformal projections. This allows the fulfillment of the Cauchy-Reimann conditions and permits the formation of a near infinite number of unique conformal projections when the constants  $A$  and  $B$  in the expansion are changed. Laborde applied this transformation to a transverse Mercator projection of a conformal sphere for the mapping of the island of Madagascar. He expanded the polynomial in Figure 17 to three terms (making  $n$  in the equation above equal 3) and produced a low error conformal projection. The projection had its central line aligned with the oblique geographic orientation of the island rather than with the central meridian of the transverse Mercator. In this way he produced a custom low-error projection for a specific region.

The disadvantage of this type of transformation is that the length of the series expansion, and hence the computational time required, grows rapidly as the irregularity of the lines of constant scale increase. In theory, the variables  $A_j$ ,  $B_j$ , and  $n$  in Figure 17 can be changed to make the scale factor on the new projection follow almost any prescribed pattern to minimize distortion in certain regions of the map. In practice, the problem is soluble only in special cases, and as Laurence Lee pointed out in his groundbreaking paper on the computation of conformal projections, "there would appear to be only a slim hope of making the scale constant along any desired curve, however simple such a curve may appear."<sup>17</sup> Lee showed that there are limits to the shapes that can be projected using this technique and that not every geographic area can be surrounded by a custom-made curve of minimum scale error.

John Snyder began to explore the implications of series expansions that fulfill the conditions of the Cauchy-Reimann equations when he attempted to solve the problem of producing minimum-error conformal map projections for irregularly shaped regions. Snyder published several papers (1984a,b) on the subject, one that addressed minimum-error projections bounded by polygons, and another that developed a low-error projection for the fifty states. In his paper on polygonal boundaries, Snyder modified a stereographic projection using the above series expansion (Figure 17). He described a procedure by which a conformal map is bounded by a line of constant scale, the area enclosed by this line closely approximating a polygon. The advantage of this series approach is that a geometric figure such as a polygon can be used to enclose a region on any conformally projected map to produce a low-error region of constant scale. The theory of enclosing a region in a line of constant scale to produce a minimum error map dates back to Pafnutiy Tschebyshev's work in 1856.<sup>18</sup> Dmitry Grave formally proved the fact that this enclosure produced the best possible map in 1896.<sup>19</sup> Snyder was able to generalize the computational methods that he worked out in this paper in order to provide closed polygons that would enclose almost any region whose display required low-error and uniformity of scale.

In the second paper Snyder developed a conformal map projection for the fifty states, which he called the GS50 projection. The paper is a brilliant example of mathematical cartography and is one of the densest and most beautifully written of Snyder's career. Snyder's original manuscripts for his various attempts at reforming known conformal projections to fit the area of the fifty states survive in his papers; an example is shown in Figure 18. He used his TI-59 calculator to modify an oblique stereographic projection and calculated the necessary coefficients in the series expansion to plot distortion and scale factors. The program runs to over four hundred lines, a portion of which is shown in Figure 19. Snyder expanded the polynomial to twenty terms and used a least squares method on forty-four points on the map. The least squares method which is an interpolation and regression technique, allowed Snyder to solve his tenth order polynomial expansion for a limited number of points and to interpolate curves between them. He changed the chosen points several times to derive the necessary shape of the low-error area he calculated.

The procedures that Snyder employed were extremely slow and time consuming. His program to calculate points of scale distortion for the GS50 takes over a minute to solve the equations

for each point. Snyder plotted the results of his calculations by hand and they are shown in Figures 20 and 21. An examination of this map reveals that the United States is shown in the only area of true conformality, in a region that displays a scale factor of 1. The map does not, however, meet Tschebyshev's criteria because the area of interest is not surrounded and enclosed by a region of constant scale; rather there are eighteen spikes that surround the region of interest. An interesting region exists in the top right corner of the map (Figure 21) where the negative forty line turns in on itself. This inversion is a common but rarely displayed feature of higher order conformal complex transformations and does not appear in Snyder's published version. The graticule for this projection is also not symmetrical about any axis, a feature that differentiates it from most common projections. The map does not provide the theoretically optimal solution; however, it is computationally tractable. A larger expansion leads to too much variation in scale between the chosen points and a smaller expansion does not provide the desired accuracy. Snyder employed Knuth's algorithm to evaluate the complex expansion and to save computational time. Because the equations were nonlinear and therefore not analytically solvable, Snyder used the Newton-Raphson method to evaluate the polynomial coefficients.

For Snyder, who was an early advocate of computer use in cartography,<sup>20</sup> the GS50 provided several interesting lessons. It showed that computers had reached a level whereby they could handle complex transformations and aid in the design of projections that had formerly been computationally intractable. Second, it showed how numerical methods could provide a deep understanding for projections whose exact analytical solution might be unknown. Computers and pocket calculators had opened up new areas of research in the field of map projections that previously were too complicated for hand computation. Solutions involving hundreds and even thousands of points became possible using new algorithms, and the iteration of large numbers of simultaneous equations was made possible by faster processing speeds in the 1970s and 1980s.

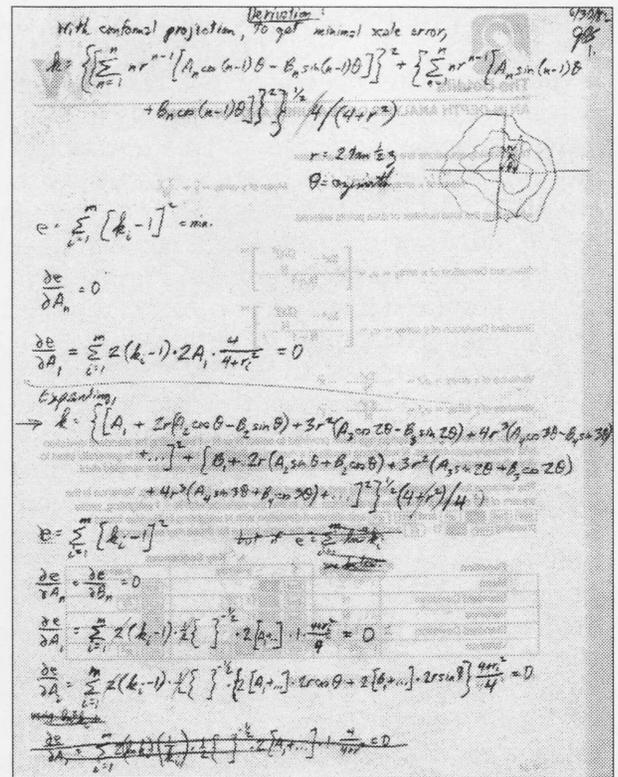


Figure 18. Manuscript of series expansion

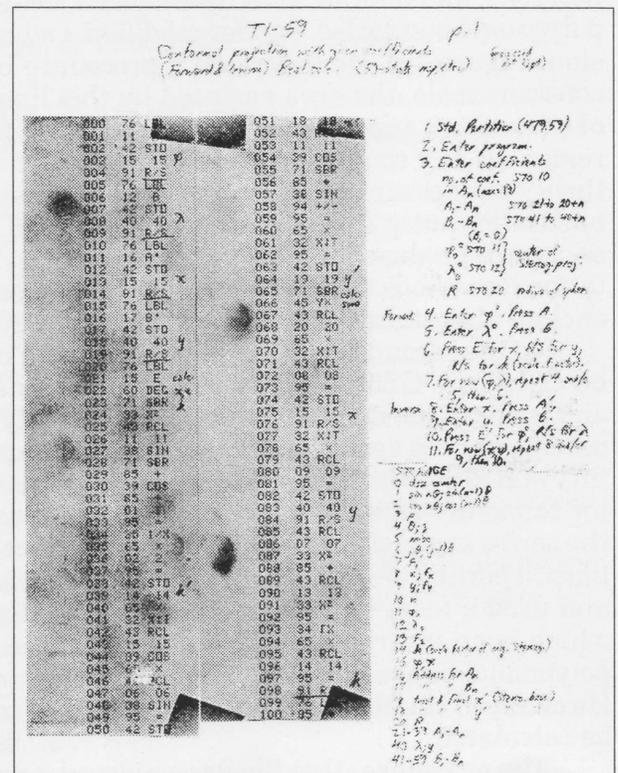


Figure 19. Conformal transformation program

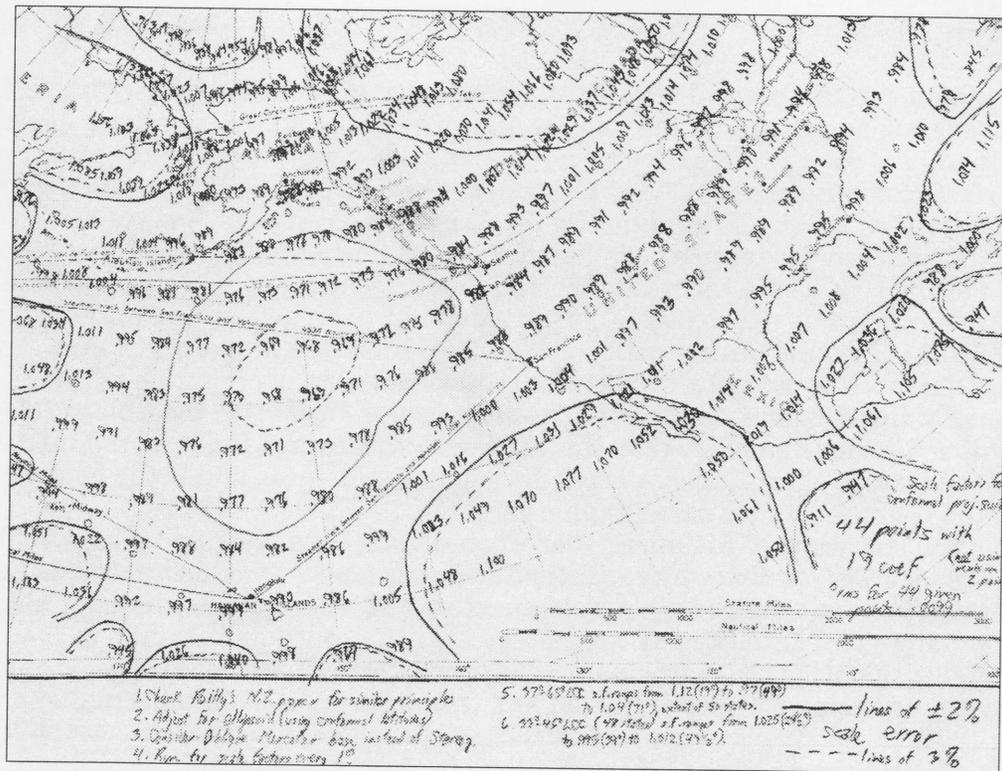


Figure 20. Distortion at selected least-squares points for GS50 projection

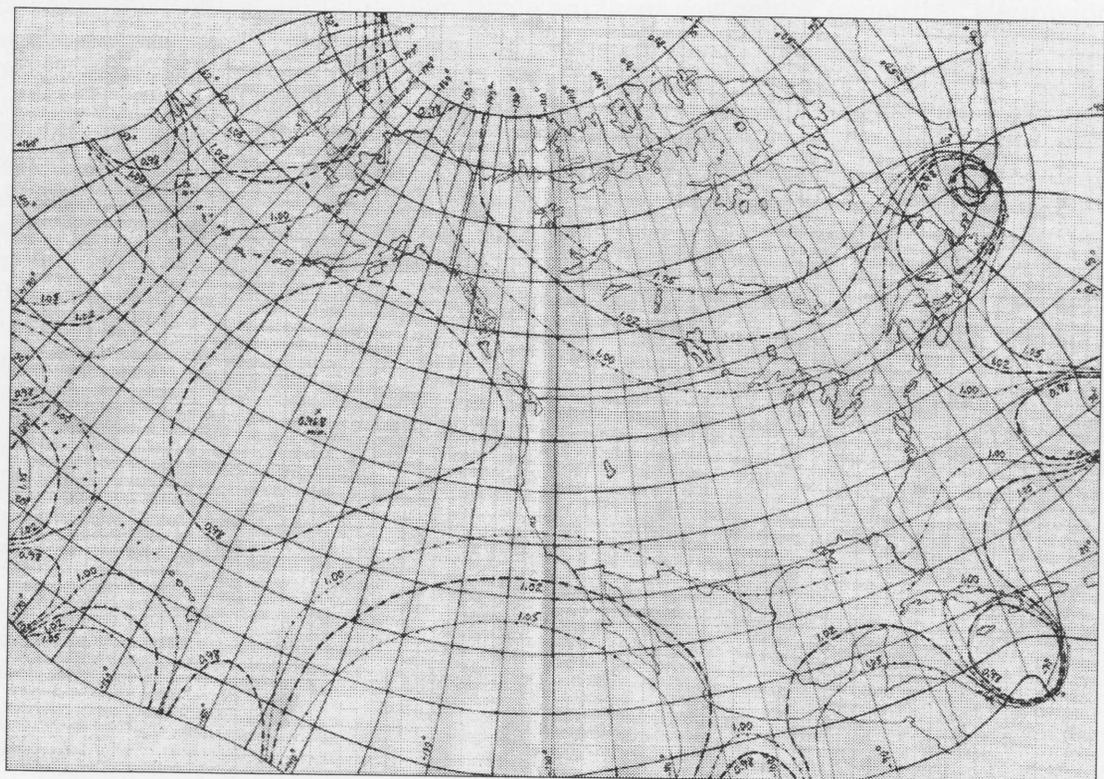


Figure 21. Smoothed distortion plot for GS50 projection

## Conclusion

Throughout the 1980s and 1990s Snyder continued to produce and publish technical articles, book reviews, and books on the subject of projections, and was involved in many professional cartographic societies. He became more and more adept at computer programming and wrote a book on computer assisted map projection research in 1985 that is still useful today. He produced his *Album of Map Projections* on an Atari home computer programming in Fortran and Basic. His computational methods found their way into the General Cartographic Transformation Package (GCTP), a software program produced by the USGS that served as the basis for commercial programs developed by Intergraph and the Environmental Systems Research Institute (ESRI). He collaborated with colleagues in Russia, Europe, and China, and through his writings did much to promote a broader, international view of projection research. His career could be said to have culminated with the publication of *Flattening the Earth: Two Thousand Years of Map Projections*, published by the University of Chicago Press in 1993. The book is an account of the history of projection science for popular audiences and is the book he envisaged writing at the beginning of his cartographic career in 1976.

During the last years of his life Snyder worked with Qihe Yang and Waldo Tobler on *Map Projection Transformation: Principles and Applications*, which was published in 2000, three years after his death. The book reflects the developments in projection science brought about by more modern computers and satellites. It concentrates on the concepts inherent in map projection transformations, updating the mathematics of projections to fit the needs of geographic and spatial information systems. The book is an ending tribute to Snyder's intellectual and publishing career.

Waldo Tobler begins the introduction to his translation of Johann Heinrich Lambert's *Notes and Comments on the Composition of Terrestrial and Celestial Maps* with the words, "The subject of map projections has seen remarkable contributions from many remarkable individuals." Although Tobler was speaking about mathematicians and cartographers such as Lambert, Euler, Gauss, and Mercator, it is now possible to include in his list the name of John P. Snyder. Snyder's solution of the equations for the Space Oblique Mercator Projection and his contributions to the mathematics of projections and coordinate transformations make him one of the most important mathematical cartographers in the long history of projection science. Although no projection or set of equations carries Snyder's name, a consequence more of his personality than of his achievements, his impact will be felt far into the future of cartography. The collection of his papers, now preserved in the Geography and Map Division, is unique and will give future researchers a glimpse into the mathematics and working methods of an extremely important cartographer at a pivotal time in cartographic history.

It has been said that all worthwhile and useful map projections have already been developed. Even Arthur Robinson in his classic book *Elements of Cartography* wrote that, "present [and future] cartographers need to devote little time to devising new projections but rather would do better to become more proficient in selecting from the ones available."<sup>21</sup> Snyder's work on the SOM proves that this is not always true and that future needs and problems cannot always be predicted. When Snyder died in 1997, he was working on equations that would help calculate the pixel distortion caused by georectification and re-projection of raster images using remote sensing data in geographic information systems. Always looking to the future and for new problems, he truly was one of Tobler's remarkable men.

## Endnotes

- <sup>1</sup> Ludwig Wittgenstein, "Remarks on Logical Form," *Proceedings of the Aristotelian Society* 9 (1929): 162–71.
- <sup>2</sup> Mark Monmonier, *New York Times Magazine*, January 4, 1998, p. 25.
- <sup>3</sup> *American Cartographer* 1 (1974): 89–90.
- <sup>4</sup> Marie Armand Pascal d'Avezac-Macaya, "Coup d'oeil historique sur la projection des cartes géographie," *Bulletin de la Société Géographie*, Series 5, 5 (1863): 257–361; and Charles Alphose Adrien Germain, *Traité des projections des cartes géographiques representation plane de la sphère et du sphéroïde* (Paris: 1865).
- <sup>5</sup> John P. Snyder, "A Comparison of Psuedocylindrical Projections," *American Cartographer* 4 (1977): 59–81.
- <sup>6</sup> Alden Colvocoresses, "Space Oblique Mercator Projection," *Photogrammetric Enginneering* 40 (1974): 921–926.
- <sup>7</sup> Private conversation between the author and Dr. Alden Colvocoresses, March 2003.
- <sup>8</sup> Alden Colvocoresses, "Planimetric Mapping from Spacecraft," pp. 352–58 in Urho A. Uotila, ed., *The Changing Face of Geodetic Science* (Columbus: Ohio State University, Department of Geodetic Science, 1977).
- <sup>9</sup> Letter in correspondence box 3, John Snyder Papers.
- <sup>10</sup> Private conversation between the author and Dr. Alden Colvocoresses, March 2003.
- <sup>11</sup> John P. Snyder, *Space Oblique Mercator Projection: Mathematical Development*, Geological Survey Bulletin 1518 (Washington D.C.: 1981).
- <sup>12</sup> Martin Hotine, "The Orthomorphic Projection of the Spheroid," *Empire Survey Review* 8 (1946): 300–11; 9 (1947): 25–35, 52–70, 112–123, 157–166.
- <sup>13</sup> John L. Junkins, *Formulation of a Space Oblique Mercator Map Projection*, Final Report no. 525023 (Ft. Belvoir, Va.: Computer Science Laboratory, US Army Engineer Topographic Laboratories, 1977).
- <sup>14</sup> John Noble Wilford, *New York Times*, May 20, 1978, p. D1.
- <sup>15</sup> John P. Snyder, "A Low-Error Conformal Map for the 50 States," *American Cartographer* 11 (1984): 27–39.
- <sup>16</sup> Ludovic Driencourt and Jean Paul Laborde, *Traité des projections des cartes géographiques* (Paris: 1932).
- <sup>17</sup> Laurence P. Lee, "The Computation of Conformal Projections," *Survey Review* 22 (1974): 245–256.
- <sup>18</sup> Pafnutiy L'vovich Tschebyshev, "Sur la construction des cartes géographiques," *Bulletin de l'académie impériale des sciences* 14 (1856): 257–61.
- <sup>19</sup> Dmitry Aleksandrovich Grave, *Ob osnovnykh zadachakh matematicheskoy teorii postroyeniya georaficheskikh kart* [Basic problems of the mathematical theory of geographical map construction] (St. Petersburg: Academy of Science, 1896), p. 231.
- <sup>20</sup> John P. Snyder, *Computer-Assisted Map Projection Research*, Geological Survey Bulletin 1629 (Washington D.C.: 1985).
- <sup>21</sup> Arthur H. Robinson, *Elements of Cartography* (New York: John Wiley and Sons, 1953), p. 202.

## Annotated Inventory and Description of the John Parr Snyder Papers

### Series I Correspondence

John Snyder's correspondence forms an extensive portion of his papers and is the most complete source of materials relating to his life. The subject matter is extremely broad as is the time frame of the letters' composition. The letters extend from 1948 through 1997, the year of his death. Included are letters to Senators, Representatives, restaurant owners, and local officials regarding the civil rights movement and the rights of African Americans. The letters document his professional engineering career and his years as a mathematical cartographer. The correspondence with Waldo Tobler on the early use of computers in mathematical cartography and Arthur Robinson on the Peter's Projection are especially important. Notes and mathematical derivations of projections appear in his correspondence, many of which are not included in his mathematical manuscripts. His interactions with the physicist Erik Gafarend on the subject of projection analysis using differential geometry deserve mention as they provide insight into Snyder's attempts to generalize some of his projection mathematics for the solution of other problems. The organization of the correspondence reflects somewhat the order in which Snyder left them.

#### Box #      Contents

- |    |  |
|----|--|
| 1  | <b>Relating to Map Projections from Various Correspondents, A–K</b>  |
| 2  | <b>Relating to Map Projections from Various Correspondents, L–Sp</b>   |
| 3  | <b>Relating to Map Projections from Various Correspondents, Sq–Z</b>   |
| 4  | <b>Relating to Published Materials</b><br><i>The Story of New Jersey Civil Boundaries</i><br><i>Map Projections for Very Large Quadrangles</i><br><i>Map Projections Used by the USGS</i><br><i>Album of Map Projections</i><br><i>Map Projections—A Working Manual</i><br><i>Flattening the Earth</i> |
| 5  | <b>Relating to Published Materials</b><br><i>New Jersey Maps and Mapmakers</i><br><i>Map Projections Transformations</i>   |
| 6  | <b>Relating to Cartographic Studies</b><br>Gnomonic Projection<br>Space Oblique Mercator<br>Mapping of New Jersey<br>Widmer's Battle Map   |
| 7  | <b>Social and Personal</b><br>Religious Organizations<br>Racial Discrimination<br>Other Political Subjects and Correspondents<br>Misc. Personal A–Z  |
| 8  | <b>Correspondence with Arthur H. Robinson</b>  |
| 9  | <b>Miscellaneous Technical</b>   |
| 10 | <b>Foreign Language Correspondence</b>   |

## Series II Mathematical and Projection Studies

The manuscripts containing mathematical and projection studies form the centerpiece of the Snyder collection, providing firsthand evidence of his mathematical techniques and his growth as a mathematical cartographer. Snyder left copious notes and saved his successes as well as his mistakes and false starts, all of which provide unique insight into his derivations. Many of the papers are extremely detailed in their analysis, and this makes them an extremely rare record of a mathematical cartographer at work. The series has been divided by projection, but throughout these notes there are many new numerical techniques that Snyder used to produce his projection studies.

### Box # Contents

- |    |  |
|----|--|
| 11 | <b>Projection Notes and Derivations</b><br>Conformal Oblated Stereographic<br>Conformal Complex Transformations (several)<br>Conformal Peirce Projection<br>Conformal Bounded Polygonal<br>Conformal Conic<br>Space Oblique Mercator                     |
| 12 | <b>Projection Notes and Derivations</b><br>Transverse Mercator<br>Polyconic Projections (several)<br>Triaxial Ellipsoid<br>Polyhedral Projections (several)<br>Perspective Formulas for an Ellipsoid<br>Interrupted Goode Homolosline<br>Dyer-Equal Area |
| 13 | <b>Projection Notes and Derivations</b><br>Equidistant (several)<br>Equal-Area Projections (several)<br>Geodesics<br>Rhumb Lines<br>Distortion Analysis<br>General Satellite Projections   |

## Series III Published and Unpublished Writings

The series of published and unpublished writings contains manuscripts dating from an early period in Snyder's life and continue through writings published after his death. His unpublished technical miscellany and unpublished manuscript on the history of projections are extremely important in that they show Snyder at the early stages of his projection work. These unpublished manuscripts foreshadow his future work on the history of projections and contain many annotations and corrections. The manuscripts of his published works include early drafts, corrected versions, and publisher proofs.

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15	<b>Published Manuscripts</b> Article "Map Projections in the Renaissance" for <i>History of Cartography Map Projections—A Reference Manual</i> Original Artwork for <i>Flattening the Earth</i>
16	<b>Reviews and Journal Publications</b> Book Reviews Projection Articles in Peer Review Journals Commentaries and Editorials
17	<b>Unpublished Manuscripts</b> Early Literary Works, Poems, Plays Notebooks Unpublished Reviews and Commentary
18	<b>Unpublished Manuscripts</b> Technical Miscellany (1947–48) Map Projection History and Applications (1976)

<b>Series IV Computer Programs</b>
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Snyder's computer programs form a difficult but extremely valuable part of this collection and present challenges to the researcher. The programs are written in many languages and utilizing a wide variety of machines. The most important of the programs are written for the TI-56 and TI-59 calculators. Snyder performed much of his most creative work on these calculators, storing his programs on magnetic strips. The code for many of these programs has been restored and can be run either on modern TI calculators or with emulation programs.

<b>Box #</b>	<b>Contents</b>
19	<b>Projection Programs Basic and Fortran</b> USGS Bulletin 1629, Computer Assisted Map Projection Space Oblique Mercator Projection Calculation of Geodesics Jacobian Elliptic Integrals
20	<b>Projection Programs Basic and Fortran</b> Map Projections from Graticules Geodesics on an Ellipse Conformal Projection Coefficients Polynomials for Direct Transformation Distortion for Oblated Equal-Area Projection
21	<b>TI-56 and TI-59 Programs for Projection Calculations</b> Azimuthal Airy's Projection Two-point Equidistant Albers Conic Equal Area Lambert Conic Conformal Equidistant Conic Young's Minimum-error Conic Chamberlin Trimetric Polyconic Transverse Polyconic Maurer's Equal-Area Polyconic Mollweide Eckert IV and VI Briesemeister Oblique Hammer Bonne Werner Lee's Conformal Miller Prolated Oblated Equal-Area

Mercator  
 Polar Stereographic  
 Conformal Latitude  
 Space Oblique Mercator  
 Miscellaneous

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22	<b>American Cartographic Society Materials</b>
23	<b>Professional Society Materials</b> Washington Map Society National Geographic Society
24	<b>George Mason University Teaching Materials</b> Lecture Notes, Evaluations, and Student Papers
25	<b>Papers Relating to Mapsat/Landsat Program Materials</b>

#### Series VI Engineering Materials

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26	<b>Engineering Materials</b> MS Thesis, Silver-Hydrogen Exchange Equilibria (1948) Engineering Publications Miscellaneous MIT materials

#### Series VII Reference Reprints

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27–31	<b>Reprints Relating to Map Projections</b>
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#### Series VIII Miscellaneous

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34	Miscellaneous Games Created by Snyder, Slide Rule, etc.
35	Three Portraits of John P. Snyder from 1965–67
36	TI-56 and TI-59 Calculator with Magnetic Tape Programs
37	<b>Celestial Globe Made by John P. Snyder in 1950</b> Truncated Icosahedron Globe Based on Bayer's Uranometria, 1603

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